bias V–I curve and calculate $R_r = \Delta V/\Delta I$ from that tangent line. Report R_r and R_rA.

5. *Reverse Biase Breakdown Voltage.* The maximum voltage a diode will take in the reverse direction is another measure of the "health" of the diode and should be noted. Although it is clear that breakdown is occurring, the definition of V_b is not universal. You may define it as the voltage at which the slope is twice that in the reverse straight-line segment, or where the slope or current reaches some predetermined threshold. Indicate V_b and record it on your report.

5.2.3. *V–I* Data for Photoconductors

Although V–I curves can be drawn for photoconductors, it is not common do so. Instead, one measures or is given one or two voltage, current pairs, or may only have the resistance values: $R = V/I$. These measurements are often made at the blackbody test set while measuring signal and noise; the primary purpose is to determine and document the bias conditions that maximize S/N.

Although not a normal part of testing, it is instructive to acquire and plot V–I data for a photoconductor over a wide range of currents and voltages. The V–I curve is much more nearly linear than for the PV detectors, but deviations from linearity do occur and their presence can be a reassuring indicator that the device is behaving as expected (see Figure 2.11). These variations come about because the mobility and lifetime of semiconductors are somewhat dependent on the electric field. (Ohm's law is, properly speaking, not truly a law. It is a definition of resistance and a statement that for many cases, resistance is independent of electric field.)

5.3. NOISE

Noise measurements are essential ingredients of detector characterization. Noise is defined as the rms deviation of the detector output from its average value. It can be determined from a number of digital samples of the detector output, from an analog meter connected to the output, or even estimated from a visual examination of an oscilloscope trace.

5.3.1. Digital Calculation

Noise can be calculated from digitized samples of the detector output: It is the standard deviation of the population (σ) of the detector output voltage (or current). Several algorithms for computing the standard deviation are available. These are usually implemented in software or firmware.

Direct Method. One way to determine noise is to apply the definition directly. First, calculate the sum of all the values, and the mean. Then go back through the list of values, calculating the deviation from the mean, the squares of the deviations, their sums, and the square root of the sum divided by $J - 1$:

STANDARD DEVIATION OF THE POPULATION

$$\sigma = \sqrt{\frac{\sum^{J} (x_j - \bar{x})^2}{J - 1}} \tag{5.1}$$

where

$$\bar{x} = \frac{\sum^{J} x_j}{J} \tag{5.2}$$

This process could be called a brute-force, or two-pass method. It is straightforward but requires that we save or store all the values, then go through the list of values twice: once for the mean value, once for the deviations. In some situations this can be undesirable; it may take too much time or too much data storage space. Once you have computed the noise, the decision to include a few more values requires you to start over and repeat the entire process with your new (larger) data set. Methods are available that avoid this problem.

Sum and Sum of Squares. The standard deviation definition can be manipulated to provide an algebraic expression that requires only the sum and sum of the squares.

STANDARD DEVIATION FROM "SUM AND SUM OF SQUARES"

$$\sigma^2 = \frac{\sum^{J} x_j^2 - (\sum x_j)^2/J}{J - 1} \tag{5.3}$$

This means that we do not have to store all the individual values—just keep a running total of x and x^2 (and the number of samples summed). Data acquisition systems can do this easily. When enough samples have been summed, calculate the noise directly. (The required number of samples is discussed in Section 5.3.2.)

The advantages of the sum and sum of squares method are that little storage space is required and the calculation is fast. Additional samples can be included without starting over. The only limitation is that the sum of the squares can sometimes become a vary large number, overflowing the available data storage capacity.

Sum and Sum of Squares, with an Offset. If we can estimate ahead of time what the average value will be, we can offset all values by that estimate before calculating the sum and sum of squares (Equation 5.4). This allows us to work with smaller numbers, perhaps avoiding the overflow problem. The offset can be added back to the average value, and the noise needs no correction since the deviation is unaffected by a constant offset.

STANDARD DEVIATION WITH AN OFFSET

$$\sigma(x) = \sigma(x') \tag{5.4a}$$

$$\bar{x}_j = \bar{x}'_j + C \tag{5.4b}$$

where

$$x'_j = x_j - C \tag{5.4c}$$

$$C = \text{offset used} \tag{5.4d}$$

This is a useful technique, even when calculating means and sigmas by hand. One limitation is that it requires some estimate of the final mean, and that inevitably involves some loss in generality in the process.

5.3.2. Error in Digital Noise Measurements

When digital techniques are used, the analog detector voltage or current is sampled, then converted to a digital value which is then processed. In addition to any systematic (calibration) error in the analog circuit, error is introduced by the digitization process (quantization error), and in the computer itself (round-off error). In any noise determination (digital or analog) there is error due to the fact that a finite data set provides only an *estimate* of the noise associated with the entire population of output values. In the following sections we discuss these three sources of error.

Round-off Error. Because computers represent all numbers in binary, there is some rounding error in all input, intermediate, and reported values. If many repeated calculations are performed, the rounding errors can accumulate. This can be a limiting contribution to noise accuracy, and in fact, one can reach the point where acquisition and processing of more and more samples, in an attempt to improve accuracy, can actually *reduce* the accuracy. Analysis of these computer related errors and algorithms to minimize them are found in articles by Hanson (1975), Chang and Lewis (1979), and West (1979) in *Communications of the Association of Computing Machinery* (ACM).

Quantization Error. When digital information is required, an analog-to-digital (A–D) converter is used. There is some error introduced in the process—the output of the A–D is a discrete value, not exactly equal to the analog (input) value. A measure of this potential error is the resolution of the digitizer, normally called the *least significant bit* (LSB). If the digitizer covers a range R (either zero to R, or from $-R/2$ to $+R/2$) with K bits, the LSB is given by

$$\text{LSB} = \frac{R}{2^K} \tag{5.5}$$

For example, for 12 bits, 2^K is 4096, so a 12-bit system can represent a 10-V range with a resolution of 2.44 mV:

$$\text{LSB} = \frac{10\ \text{V}}{4096} \simeq 2.44\ \text{mV}$$

Digitization introduces a signal error whose absolute value is between zero and LSB/2.

If the LSB is less than about two times the detector noise being measured, the effect on noise is to add an effective noise of magnitude $\text{LSB}/\sqrt{12}$ (about 0.3 LSB) to the actual noise. This can be derived by calculating the rms deviation in the signal due to the digitization process. If LSB is small compared to the noise, the probablility of any signal between $+\text{LSB}/2$ and $-\text{LSB}/2$ is roughly constant, and the rms error (or added effective noise) is

$$(\text{rms error})^2 \simeq \frac{\int_{-\text{LSB}/2}^{+\text{LSB}/2} \epsilon^2\, d\epsilon}{\int_{-\text{LSB}/2}^{+\text{LSB}/2} d\epsilon} = \frac{\text{LSB}^2}{12}$$

For LSB values that exceed about two times the detector noise, quantization can distort the data in a way that is difficult to predict. The resulting noise can be too small—as small as zero—or too large—as large as LSB/2: If the mean value is near one of the allowed digital values, the observed noise is too small (Figure 5.3a), but if the mean value is near the boundary between two digital ranges, the reported noise will be too large (see Figure 5.3b).

EFFECTIVE NOISE DUE TO QUANTIZATION ERROR

$$\text{Quantization noise} \simeq \frac{\text{LSB}}{\sqrt{12}} \quad \text{if LSB} < 2 \times \text{anticipated basic noise} \tag{5.6}$$

Quantization noise becomes more unpredictable as LSB exceeds the basic noise; it can be as large as LSB/2, or can be zero.

Effect of Sample Size on Error in Noise Measurements. If we calculate the noise from J independent (as defined below) signal values, using the rms value of the deviations from the average, as described earlier, then the uncertainty (1 σ, or 1 standard deviation) for that noise value is given by Mandel (1964, p. 236), as seen in Eqs. 5.7 and 5.8.

Table 5.2 Uncertainty in Noise Measurements for Different Numbers of Independent Samples

Number of Samples	Probability (%) That Noise Is within Specified Percentage of "Correct" Value		
	68%	95%	99.7%
10	22	45	67
30	13	26	39
100	7	14	21
300	4	8	12
1000	2	5	7
3000	1	3	4

For 100 samples there is a 68% probability that the result is within 7% of the "correct" values. It requires 3000 samples for 95% confidence that the value is within 3% of the "correct" value.

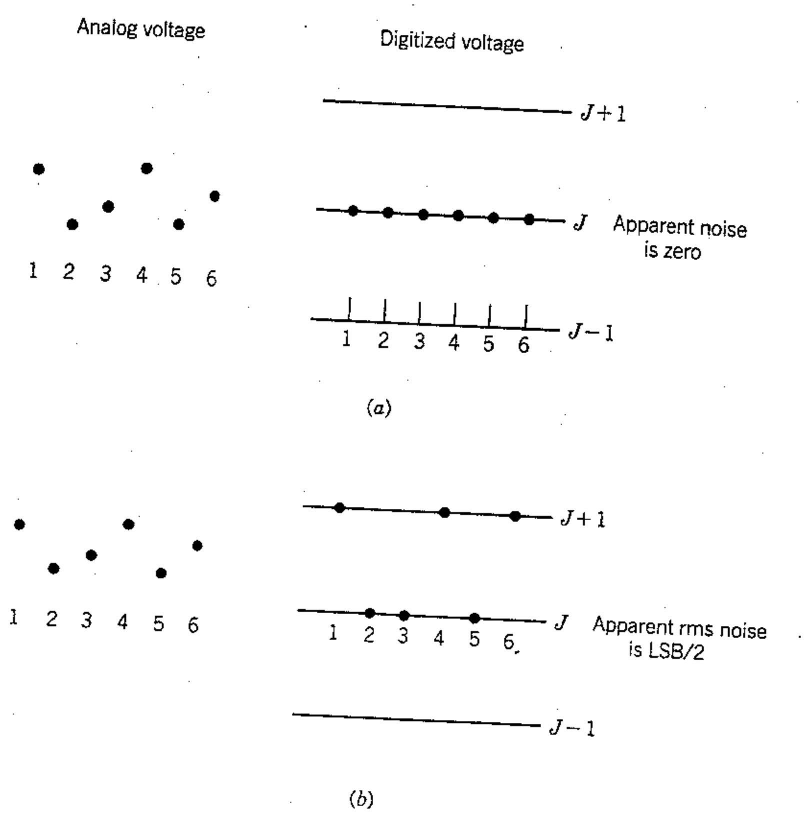

Figure 5.3. Large quantization effects mask actual noise; (a) overage near one digital level; (b) average between digital levels.

UNCERTAINTY IN NOISE DUE TO FINITE NUMBER OF SAMPLES

$$\sigma_{\text{noise}} = \frac{\text{noise}}{\sqrt{2J}} \tag{5.7}$$

UNCERTAINTY EXPRESSED AS A RELATIVE NOISE ERROR

$$\frac{\sigma_{\text{noise}}}{\text{noise}} = \sqrt{\frac{1}{2J}} \tag{5.8}$$

The 1, 2, and 3σ values are given in Table 5.2 and Figure 5.4. The table and figure are based on the formula above for the standard deviation of noise values and the normal curve of error (discussed in Chapter 6).

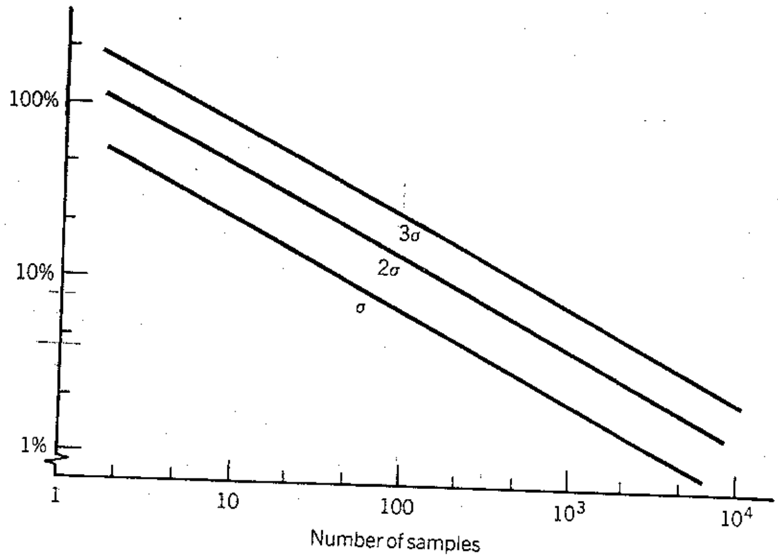

Figure 5.4. Relative uncertainty in noise versus sample size.

Example 1: If noise is determined from the rms deviation of 100 samples, there is a 68% probability (sometimes referred to as 1σ probability) that the result is within 7% (Table 5.2) of the "true" value, and a 95% probability that the result is within 14% of the "true" value (Figure 5.4). ■

Example 2: To be 95% certain (i.e., 2σ) that the noise is within 1% of "correct" requires about 20,000 samples (Figure 5.4). ■

Mandel (1964) comments that "it is a deplorable misconception, based on ignorance or disregard of elementary statistical fact, that small samples are sufficient to obtain satisfactory estimates of variability."

The noise uncertainty expressions given above assume that normal or gaussian statistics apply, and that the samples are independent. White noise obeys gaussian statistics, but other types may not. $1/f$ noise is one that does not.

Samples are independent as long as they are taken at a rate less than $2\Delta f$, where Δf is the noise equivalent bandpass of the system. If the signal is measured by an analog device that integrates for time t, the number of independent samples is $t \times 2 \times \Delta f$, and the integration time required to include n independent samples is $t = n/(2\,\Delta f)$.

Example: With a 20-Hz equivalent noise bandwidth we can take independent data no faster than 40 samples per second. To acquire 20,000 samples would take 500 s, more than 8 min. ■

5.3.3. Analog Measurements of Noise: Meters and Their Errors

Analog meters with which one could attempt to measure noise fall into three general types:

Meter 1 simply averages—it is called an *average responding meter.* It is of no use to us as a noise meter; it always reads the average value—either just the signal, or zero if there is no signal.

Meter 2 actually averages the square of the wave, then takes the square root—it is a *true rms meter.* It is ideal, and we should always use such a true rms meter if we can get it.

Meter 3 averages a rectified wave. It can be used, but will read gaussian noise low by a factor of $2/\sqrt{\pi}$ (about 1.128). This occurs because a meter like this is generally calibrated so that it displays directly the true rms value of *sinusoids.* To do this, the scale is "fudged" to display a value $\pi\sqrt{2}/4$ (about 1.11) times the average of the full wave rectified deviation. The rms of noise with a gaussian distribution is $\sqrt{2\pi}/2$ (about 1.25) times greater than the average of the rectified deviation. Thus readings gaussian noise made with such a meter must be multiplied by an additional factor of $2/\sqrt{\pi}$ (about 1.128).*

5.3.4. Bandwidth for Noise Calculations

The bandwidth of electrical circuits affects the noise that passes through the circuit. For a perfect square filter the bandwidth is clear, but for a filter that "turns on" and "off" more gradually, the equivalent noise bandwidth must be calculated from the circuit gain as a function of frequency. This equation is given by Hudson (1969) in terms of the power gain $G(f)$, and by Jones et al. (1960) in terms of the voltage gain $g(f)$. It appears here as Eq. 5.9. One way to do the indicated calculation is to plot (on linear paper) the square of the voltage gain versus frequency, determine the area under the curve, and construct a rectangle of height

* Jones et al. (1960, Sec. 3.7) comments on this source of error, but states that the meter will read low by 0.9003. This is apparently an error; the 1.128 value is well documented elsewhere (Hudson, 1969. pp. 317, 318; General Radio Company, 1963, p. 4; Marconi Instruments, 1965, pp. 18–22).

g_0 and width such that it includes the same area as the curve. The width of the rectangle is the equivalent noise bandwidth (see Figure 5.5).

EQUIVALENT NOISE BANDWIDTH FROM VOLTAGE GAIN

$$\Delta f = \int_0^\infty \left[\frac{g(f)}{g_0}\right]^2 \Big/ \, df \qquad (5.9)$$

where $g(f)$ is the voltage gain at frequency f and g_0 is the maximum voltage gain.

For a circuit that rejects high frequencies at 3 dB per octave (a single-pole filter so that the response to a voltage pulse is an exponential decay with time constant τ) with a corner frequency f_0, the equivalent noise bandwidth is expressed in Equation 5.10. A circuit that integrates for time T averages out the high-frequency effects (both signal and noise). The effective noise bandwidth is given by Lange (1967) and Boyd (1983) and is shown in Equation 5.11.

EQUIVALENT NOISE BANDWIDTH: SINGLE-POLE FILTER

$$\Delta f = \frac{\pi}{2} f_0 = \frac{1}{4\tau} \qquad (5.10)$$

where $f_0 = 1/2\pi\tau$.

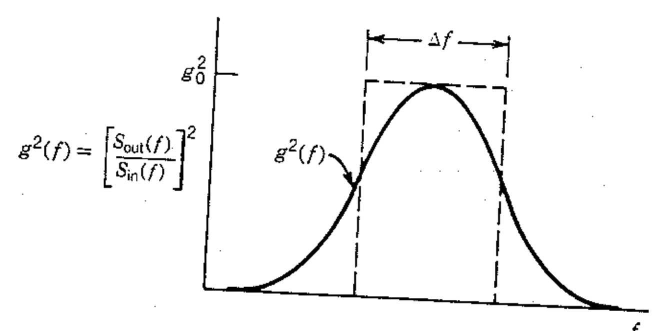

Figure 5.5. Equivalent noise bandwidth.

EQUIVALENT NOISE BANDWIDTH: INTEGRATOR

$$\Delta f = \frac{1}{2T} \tag{5.11}$$

5.3.5. Characterization of Low-Frequency Noise

Most semiconductors exhibit increased noise at low frequencies, as shown in Figure 5.6. For some devices this excess noise spectral density fits the following equations:

$$\text{power} \propto \frac{1}{f} \tag{5.12a}$$

$$\text{voltage} \propto \sqrt{\frac{1}{f}} \tag{5.12b}$$

Noise that fits those equations is *1/f noise*. (Note that $1/f$ noise ***voltage*** varies as one over the ***square root*** of frequency.) Very often we refer to the increased noise at low frequencies as $1/f$ noise—whether or not it obeys that equation.

The acceptable amount of $1/f$ noise is sometimes specified, or must be measured for other reasons. This can be done using a spectrum analyzer, which generates a plot of spectral density versus frequency. The amount of $1/f$ noise can also be conveyed by a single number—the frequency that divides the excess noise region from the white noise region. The higher this frequency, the higher the low-frequency noise will be (see Figure 5.6).

The critical frequency can be defined and determined in several ways. Since the results will not be the same (unless the noise obeys the assumed $1/f$ relationship exactly) it is important that everyone involved reach agreement on which method will be used.

The real need is *not* for $1/f$ corner frequency. Remember that the customer's ultimate concern is probably not really the $1/f$ corner frequency, but rather some system-level effect (such as flicker, or uniformity drift) which we know is related to low-frequency noise, but whose dependence on low-frequency noise is complex and generally only poorly understood. The corner frequency is just an attempt to get a handle on a more complicated problem. It probably does not make sense to spend a great deal of time refining the $1/f$ corner frequency definition until the real system need is equally well identified and characterized.

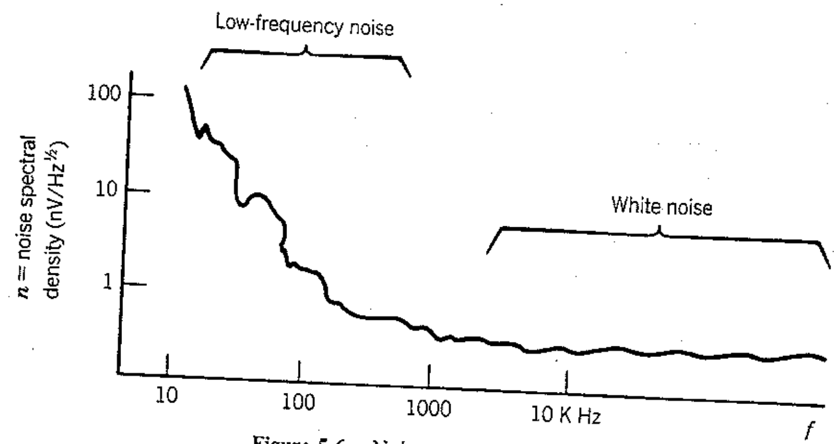

Figure 5.6. Noise spectral density.

This is particularly true since $1/f$ noise is very difficult to characterize reliably since it is by nature erratic and variable. If noise readings are required between 50 and 300 Hz the presence of 60-Hz line noise will further complicate detector $1/f$ noise characterization.

Noise Spectral Density Plots. The most complete and accurate way to characterize low-frequency noise is with a plot of noise spectral density (volts/square root hertz) versus frequency (Figure 5.6). This method suffers from two problems: noise spectrals are slow to acquire, and the data are not compact (only one or two curves fit on a page).

3-dB Frequency. We are sometimes requested to condense the low-frequency noise information into just one number. One way to do that is to provide the ***3-dB frequency*** f_{3dB}: the frequency below which the excess noise is 3 dB* above the white noise (see Figure 5.7). This definition works even if the noise does not obey the $1/f$ equation.

Determination of f_{3dB} as defined here requires narrowband noise measurements at several frequencies near the expected f_{3dB}, as well as a good fix on n_{white}. First fit a horizontal line through the high-frequency data, and note the corresponding noise spectral density. Draw a smooth curve through the low-frequency data. The frequency at which the noise is 1.413 higher than the horizontal line is the 3-dB frequency.

* "3 dB" and "a square root of 2 in voltage ratio" are often used interchangeably. Strictly speaking, 3 dB is a voltage ratio of 10 to the 3/20 power (1.4125 . . .); this is within about 0.1% of the square root of 2. For most purposes this is entirely adequate.

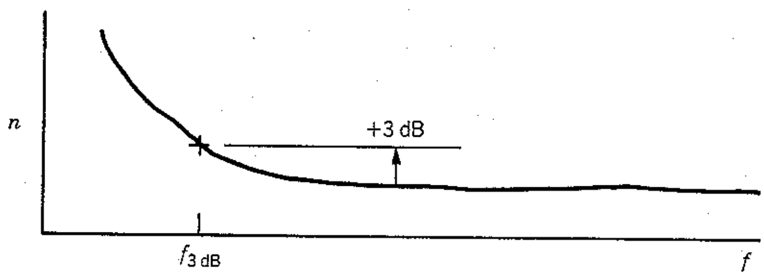

Figure 5.7. 3-dB frequency can be defined even if $n(f)$ does not obey simple equation.

Corner Frequency. Another way to characterize low-frequency noise with one number is to provide the *corner frequency* f_c. To visualize this definition (and to determine f_c graphically), plot the noise spectral density on log paper and draw two straight-line approximations: one for low frequencies and a horizontal one for the high frequency (white noise), as in Figure 5.8. The corner frequency is the frequency at which the straight-line approximations meet. This "works" if the noise can be approximated by straight lines on log paper, which is equivalent to saying that the noise obeys a power law: The square of the noise voltage varies as $1/f$ to some exponent a. (a is 1 for true $1/f$ noise.) If this is not true (see Figure 5.7, for example), the corner frequency is ambiguous and not a very useful figure of merit.

If the corner frequency can be defined, and if the noise varies smoothly, the corner frequency and the 3-dB frequency are equal: At the corner frequency the $1/f$ and white noise are equal, so the total noise spectral

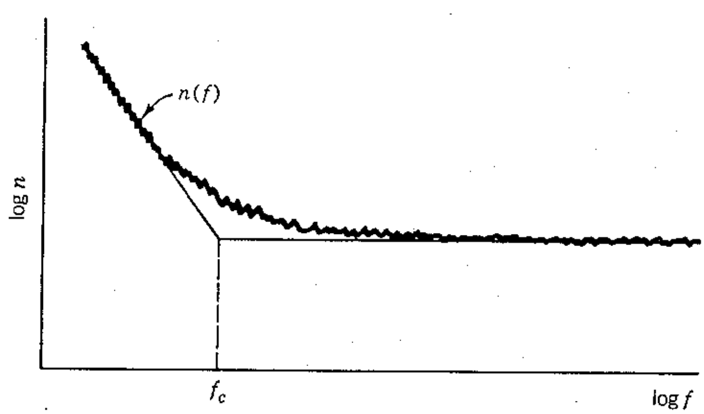

Figure 5.8. Corner frequency can be defined only if log n' versus log f is linear at high and low frequencies.

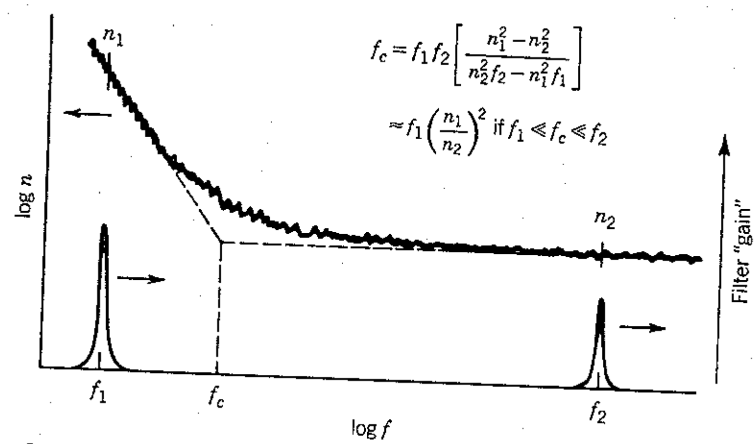

Figure 5.9. Noise measured in two narrow bandpasses allows determination of corner frequency.

density equals the square root of two times the white noise spectral density; this is 3 dB (within acceptable error).

If the foregoing conditions are met, the total noise spectral density is given by

$$n^2 = n_{\text{white}}^2 \left[\left(\frac{f_c}{f}\right)^a + 1\right] \tag{5.13}$$

where a is the exponent mentioned earlier.

Corner Frequency from Two Data Points. If the noise obeys a known power law (including, but not limited to $1/f$) the corner frequency can be determined without measuring the complete noise spectral density. This saves test time. We need only measure the noise spectral density at two frequencies, or the integrated noise in two narrow bandwidths (see Figure 5.9).

The equations are most easily derived if we know the noise spectral densities at two discrete frequencies (see Equation 5.14*a* and 5.14*b*).

1/*f* CORNER FREQUENCY FROM TWO NOISE SPECTRAL DENSITIES

$$f_c = f_1 f_2 \frac{n_1^2 - n_2^2}{n_2^2 f_2 - n_1^2 f_1} \tag{5.14a}$$

$$f_c \simeq f_1 \left(\frac{n_1}{n_2}\right)^2 \qquad \text{if } f_1 \ll f_c \text{ and } f_2 \gg f_c \qquad (5.14b)$$

where f_1 and f_2 are the center frequencies of the narrowbands, and n_1 and n_2 are the corresponding noise spectral densities.

It is not essential to measure the noise spectral density using narrow-bandpass filters. The bandpasses of Figure 5.10 could be used, for example. In that case the formulas are more complex and depend on the shape of the electrical filter characteristics. Once again, all these methods will yield different values unless the actual noise obeys our "ideal" formula.

Considerations in Selecting the Bandpasses for 1/*f* Determination

1. Integration times for any desired accuracy are less if wide bandwidths are used.
2. Equations are simpler, derivations are easier, and accuracy is better if:
   a. One measurement is limited to frequencies much less than f_c, and one much greater than f_c
   b. A narrow bandpass is used for the low-frequency filter.
3. If a narrow frequency bandpass is used, noise at discrete frequencies (60 Hz, for example) can contaminate the results easily. The

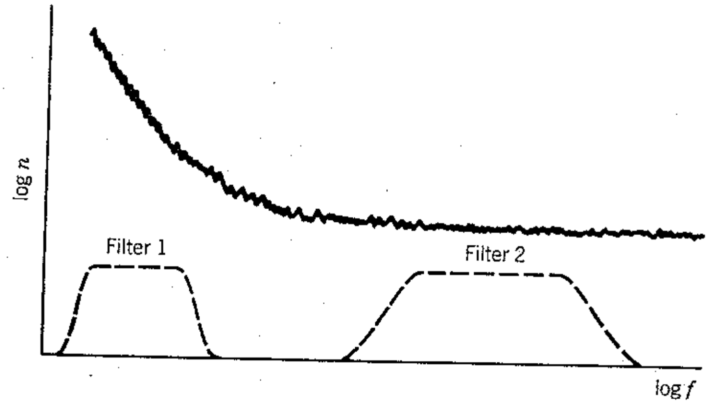

Figure 5.10. Wide bandpass electrical filters can be used to determine corner frequency, but the analysis is more complicated.

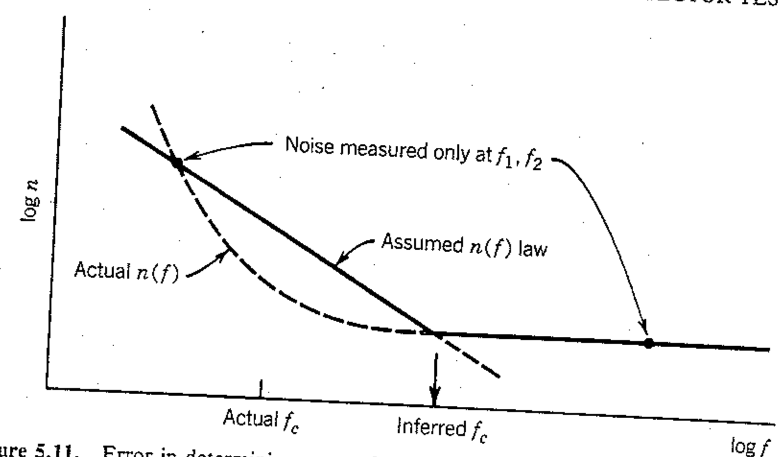

Figure 5.11. Error in determining corner frequency results if the $n(f)$ dependence is not as assumed.

wider the bandpass, the less effect a discrete frequency problem will have.

Figure 5.10 shows a filter combination that works well. The low-frequency filter has a bandpass that is about 25% of the expected corner frequency and is centered well below the expected corner frequency. The high-frequency filter has a broad bandpass but is well above the expected $1/f$ corner frequency and well below the the system cutoff frequency.

The closer the spectral density is to the assumed equation, the better different methods will agree. Generally, detectors of the same type will have similar low-frequency characteristics, and any one method will allow a useful comparison of the goodness of different detectors.

In general, however, we will agree to determine an "effective" corner frequency by applying one of these algorithms to our data, *without questioning the actual shape of the noise spectral density*. The resulting ambiguity or loss of accuracy is the price we pay for attempting to describe a spectral phenomena with just one number. See Figure 5.11 for an example of such an error.

5.4. BLACKBODY SIGNAL AND RESPONSIVITY

The most common measurement of detector performance is the blackbody signal and responsivity. The detector is placed so that it can "see" a blackbody, and the resulting signal is observed. The signal can be either the dc voltage (or current) from the bias circuit, or the ac signal that results